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Pattern Classification EET3053 Lecture 02: Feature Extraction

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Feature Extraction

■ Boundary Representation

- □ Boundary (Border) following
- \Box Chain codes
- Polynomial Approximation
- \Box Signatures
- □ Boundary Segments
- Skeletons

Boundary Descriptors

- \Box Some Simple Descriptors
 \Box Shape Numbers
- Shape Numbers
- □ Fourier Descriptors
- Statistical Moments

■ Regional Descriptors

- Texture: Moment Invariants
- □ Texture: GLCM, LBP

- Extract features which are good for classification.
- Good features:
	- \Box Objects from the same class have similar feature values
	- □ Objects from different classes have different values.
- $\;\blacksquare\;$ Bad Features: features simply do not contain the information needed to separate the classes, doesn't matter how much effort you put into designing the classifier. Results Text

$$
\begin{array}{ccc}\n & x & x & 0 & 0 & x & x \\
0 & 0 & x & x & x & 0 & 0 & x \\
0 & 0 & 0 & x & x & 0 & 0 & x \\
0 & 0 & 0 & 0 & x & 0 & 0 & x \\
0 & 0 & 0 & 0 & 0 & 0 & x \\
0 & 0 & 0 & 0 & 0 & 0 & x\n\end{array}
$$

- In general, we use labeled features for supervised learning.
- The mapping from pattern to features that is unique whereas mapping from feature vector to pattern is not immediate.
- So, many patterns may be matched to the same feature of vector.
- In pattern recognition, we never talk about a single pattern. We always talk about feature vector.

Nature of separating plane?

- For 2 dimensional feature space $-$ line
- For 3 dimensional feature space $-$ plane
- For more than 3 dimensional feature space $-$ hyperplane

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Boundary Representation

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Binary Images

Figure: Binary Images

Boundary (Border) algorithm for its boundary region of \mathbb{R}^n the *first* neighbor encountered whose value is 1, and let be the (back- $\frac{1}{2}$ boundary, consists of the following steps: ı,

- \blacksquare Assume a binary image in which object and background points are labeled 1 and 0, that is labeled 1. Denote by the *western in the see Fig. 11.1(b)* and *west* neighbor of α the object and background point **2.** Let and [see Fig. 11.1(c)]. **1.** Assume a binary image in which object and background points are
	- $\;\blacksquare\;$ Assume images are padded with the border of 0's to avoid object merging with the image border at an operation. Let denote direction in a clockwise direction. Let \mathcal{L} with the border of U´s to avoid object merging with the respectively.
■ Assume images are padded with the border of 0's to avoid ol the *first* neighbor encountered whose value is 1, and let be the (back-

c
C

property that a polygonal approximation to the boundary has a convex vertex at that location.Also, the

a b c de la constantin

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Boundary algorithm (Moore boundary tracking algorithm)

- 1. Let the starting point, b_0 be the *uppermost, leftmost* point in the image that is labeled 1. Denote by c_0 the west neighbor of b_0 . Clearly, c_0 always is a background point. Examine the 8-neighbors of b_0 , starting at c_0 and proceeding in a clockwise direction. Let b_1 denote the first neighbor encountered whose value is 1, and let c_1 be the (back-ground) point immediately preceding b_1 in the sequence. Store the locations of b_0 and b_1 for use in Step 5.
- 2. Let $b = b_1$ and $c = c_1$.
- 3. Let the 8-neighbors of b, starting at c and proceeding in a clockwise direction, be denoted by n_1, n_2, \ldots, n_8 . Find the first n_k labeled 1.
- 4. Let $b = n_k$ and $c = n_{k-1}$
- 5. Repeat Step 3 and 4 until $b = b_0$ and the next boundary point found in b_1 . The sequence of b points found when the algorithm stops constitutes the set of ordered boundary points.

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We could h[a](#page-7-0)[v](#page-8-0)[e](#page-10-0)[s](#page-12-0)[t](#page-13-0)[a](#page-14-0)[t](#page-15-0)[e](#page-16-0)[d](#page-18-0)[t](#page-20-0)[h](#page-22-0)[e](#page-23-0)[al](#page-26-0)gorith[m](#page-27-0) [j](#page-29-0)[u](#page-30-0)[s](#page-31-0)[t](#page-32-0)as easily bas[ed](#page-34-0)[o](#page-37-0)[n](#page-39-0)[f](#page-41-0)[o](#page-42-0)[l](#page-43-0)[l](#page-44-0)[o](#page-45-0)[w](#page-47-0)[i](#page-48-0)[ng](#page-50-0) a

Boundary algorithm - stopping criterion. In the counterclockwise direction rithms formulated on the assumption that boundary points are ordered in that

Figure: Illustration of an erroneous result when the stopping rule is such that boundary-following stops when the starting point, b_0 , is encountered again

Boundary representation: Chain Codes can be normalized with respect to the starting point by a straightform ward procedure: We simply treat the chain code as a circular sequence of di-

- \blacksquare In order to represent a boundary, it is useful to compact the raw data (list of $\mathsf{boundary\,\, pixels}$
- **n** Chain codes: list of segments with defined length and direction
	- □ 4-directional chain codes instead of the code instead of the chain codes instead of the code instea
	- □ 8-directional chain codes

Figure: Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code

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Boundary representation: Chain Codes $r_{\rm acc}$ rection numbers and redefine the starting point so that the resulting point so that the resulting point so sequence of numbers forms an integer of minimum magnitude. We can nor-

- \blacksquare It may be useful to downsample the data before computing the chain code a before computing the chain code instead of the chain code instead of the conde
	- to reduce the code dimension
	- \Box to remove small detail along the boundary

 \liminf Figure: (a) Digital boundary with resampling grid superimposed, (b) Result of resampling, (c) 8-directional chain-coded boundary.

 \blacksquare Can you draw 4-directional chain-coded boundary?

12.12

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se[quence of numb](#page-1-0)ers forms [an integer of minimum](#page-7-0)[m](#page-26-0)agnitude. [W](#page-27-0)[e](#page-28-0) [c](#page-29-0)[a](#page-30-0)[n](#page-31-0)[no](#page-33-0)r-

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Boundary representation: Chain Codes

Boundary representation: Differential Chain Code p_{S}

- \blacksquare The chain code of a boundary depends on the starting point. The starting point in Fig. 11.4(c) is the starting point.
	- \Box normalize with respect to the starting point (circular sequence)
	- \Box the new starting point is the one who gives a sequence of numbers giving the smallest/largest integer.
- Normalize with respect to rotation: code can be not controlled with respect to the starting point by a straightform-
	- \Box First difference can be used
	- \Box E.g., 10103322 \Rightarrow 3133030 (counting CCW) and adding the last transition (circular sequence: $2 \Rightarrow 1$) $s \to 1$
		- \Rightarrow 31330303 (Differential Chain Code) os (*Differencial Chain Code)* by using the chain code instead of the code instead
		- \Rightarrow 03033133 (Independent of starting point, i.e., rotation invariant)

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Differential Chain Code

Can you write the Differential Chain Code?

- \blacksquare Chain code: 0766666453321212
- **Differential chain code: 7700006160771716**
- $\frac{1}{2}$ for the code of the code instance, the first difference of the $\frac{1}{2}$ \blacksquare Differential chain code (rotation invariant): 0000616077171677

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Representation **Boundary Representation** Boundary Descriptors Regional Descriptors References [c](#page-17-0)ent elements of the code.ou
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Can $\frac{1}{\sqrt{2}}$ $\frac{1}{1}$ $\frac{1}{\text{i}}$ nda
⁰⁰⁰ changes (in a counterclockwise dire[cti](#page-33-0)on in Fig.tra
|-
an $\frac{F}{C}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ if $\frac{1}{\sqrt{2}}$ Feature Extraction

Differential Chain Code: Validation changes (in a counterclockwise direction in Fig.

 $\frac{1}{\sqrt{1-\frac{1$ Can you write the Differential Chain Code?

- $\frac{13}{16}$ Chain code: 0707065444442311
- Differential chain code: 7171677000061607

1

6077171677 (validated ■ Differential chain code: 7171677000061607
■ Differential chain code: 0000616077171677 (validated) $\overline{}$

> boundary. chain-coded (c) 8-directional resampling. (b) Result of superimposed. resampling grid boundary with (a) Digital **FIGURE**

rential chain code is invariant to rotation af w
er
c |
|
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| e (b) Result

References el
el e? \overline{a} ■ Differential chain code. 0000010077111077 (vanuated)
■ Is the differential chain code is invariant to rotation at any angle? (HW) Freeman chain **EXAMPLE 11.1:**

4 $\overline{ }$

the first difference of the 4-direction

Because the object of interest is embedded in small fragments,

11.3) that separate two adja-

Representation

- each pair of adjacent points defines a segment of the points defines a segment of the polygon. The goal of adj
	- \blacksquare A digital boundary can be approximated with arbitrary accuracy by a polygon.
	- In practice, the goal of polygonal approximation is to capture the "essence" of the boundary shape with the fewest possible polygonal segments.
		- \Box Minimum-perimeter polygon \mathbf{r} is to encomplementary \mathbf{r}
		- □ Splitting technique in Fig. 11.6(b). Think of the boundary as a rubber band. As it is allowed to shrink, the rubber band will be constrained by the inner and outer walls of

Figure: (a) An object boundary (black curve). (b) Boundary enclosed by cells (in gray). (c) Minimum-perimeter polygon obtained by allowing the boundary to shrink. The vertices of the polygon are created by the corners of the inner and outer walls of the gray region.

[Feature Extraction](#page-1-0) **[Boundary Representation](#page-7-0) Representation** [Boundary Descriptors](#page-27-0) Regional Descriptors [References](#page-51-0) Polygonal Approximation: Minimum-Perimeter Polygon [wi](#page-16-0)[ll](#page-17-0)[b](#page-19-0)[e](#page-20-0) [e](#page-21-0)[it](#page-22-0)[h](#page-23-0)[e](#page-24-0)[r](#page-25-0) [a](#page-26-0) *convex* or [a](#page-27-0) *[co](#page-29-0)[n](#page-30-0)[c](#page-31-0)[a](#page-32-0)[ve](#page-33-0)* vertex, with the an[gle of a vertex bei](#page-34-0)[n](#page-47-0)[g](#page-48-0)[an](#page-50-0) *interior* angle of the 4-con[ne](#page-28-0)cted boundary. Convex and concave vertices are

and concave (black dots) vertices obtained by following the boundary of the dark gray region in the counterclockwise direction. (c) Concave vertices (black dots) displaced to their diagonal mirror locations in the outer wall of the bounding region; the convex vertices are not changed. The MPP (black boundary) is superimposed for reference. Figure: (a) Region (dark gray) resulting from enclosing the original boundary by cells. (b) Convex (white dots)

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Polygonal Approximation: Splitting Technique erated in various ways. One of the simplest is to plot the distance from the cen-

 $\;\blacksquare\;$ One approach to boundary segment splitting is to subdivide a segment successively into two part until a *specified criterion* is satisfied. th a *specified criterion* is sat

Figure: (a) Original boundary, (b) Boundary divided into segments based on extreme points, (c) Joining of vertices, (d) Resulting polygon

Polygonal Approximation: Splitting Technique

- For a closed boundary, the best starting points usually are two farthest points in the boundary.
- **Farthest point can be obtained by Karhunen-Loeve transform (KLT).**
- The maximum perpendicular distance from a boundary segment to the line joining its two end points not exceed a preset threshold.
- Splitting procedure with a threshold equal to 0.25 times the length of line ab.

- **Signatures**
	- A signature is a 1-D representation of a boundary (which is a 2-D thing): it should be easier to describe.
		- e.g., distance form the centroid vs angle.

Figure: Distance-versus-angle signatures. (a) $r(\theta)$, is constant, (b) the signature consists of repetitions of the pattern $r(\theta)=Asec(\theta)$ for $0\leq\theta\leq\pi/4$ and $r(\theta)=Acsc(\theta)$ for $\pi/4<\theta\leq\pi/2$ to rotation can be achieved by finding a way to select the same starting point

Signatures

- Signatures are invariant to translation, but variant to rotation.
- Invariant to rotation: depends on the starting point \Box the starting point could be the farthest point from the *centroid*.
- \blacksquare Scaling varies the amplitude of the signature
	- \Box invariance can be obtained by normalizing between 0 and 1, or
	- \Box by dividing by the variance of the signature (does not work on circle)

- Decomposing a boundary into segments often is useful.
- Decomposition reduces the boundary's complexity and thus simplifies the description process.
- In this case use of the *convex hull* of the region enclosed by the boundary is a powerful tool for robust decomposition of the boundary.

Boundary Segments

- Convex hull H of an arbitrary set S is the smallest convex set containing S.
- The set difference $H S$ is called the *convex deficiency* D of the set S . nce $H-S$ is called the *convex deficiency* D of the
- \blacksquare Note that in principle, this scheme is independent of region size and orientation. rincipie, this scheme is independent of region size ar

Figure: (a) A region, S , and its convex deficiency (shaded). (b) Partitioned boundary.

point of a skeleton stroke and obviously should not be deleted. Deleting if it

p1

Skeletonization \mathbf{H} clidean distance. The same results would be obtained with the maximum disk of Section 9.5.7.

- One way to represent a shape is to reduce it to a graph, by obtaining its *skeleton* via thinning (*skeletonization*)
- \blacksquare MAT (Medial axis transformation) is composed by all the points which have more than one closest boundary points ("*prairie fire concept*") ${h}$ concept"). and thus satisfy conditions (c) and (d), as well as well as α \mathcal{L} (d) and \mathcal{L} (d) and \mathcal{L} (d) and \mathcal{L} (d) and \mathcal{L}

Figure: (a) Medial axes (dashed) of three simple regions,(b) Human leg bone and skeleton of the region

Boundary Features/Descriptors

- Simple descriptors
	- lacktrianglength of a boundary is one of its simplest descriptors.
		- \Box The number of pixels along a boundary gives a rough approximation of its length.
		- \Box For a chain coded curve with unit spacing:

 $length = Horizontal + Vertical + \sqrt{2} \times Diagonal$

 \blacksquare diameter (length of the major axis)

$$
\mathrm{Diam}(B) = \max_{i,j} \left[D(p_i, p_j) \right]
$$

- \blacksquare The *minor axis* of a boundary is defined as the line perpendicular to the *major axis*.
- Basic rectangle (formed by the major and the minor axis; encloses the boundary) and its

$$
\overline{\text{eccentricity}} = \frac{\text{major axis}}{\text{minor axis}}
$$

the boundary by selecting a larger grid spacing, as $\mathcal{L}_{\mathcal{A}}$ shows. Then, as $\mathcal{L}_{\mathcal{A}}$

Shape Number a chain code is independent of a chain code is in

- Shape number: the first difference as smallest magnitude (treating the chain code as a circular sequence) best approximates that of $\mathbf s$
- \blacksquare Order of a shape: the number of digits in Shape number.

grid. One way to normalize the grid orientation is by aligning the grid orientation is by aligning the chain-code

a b

a b

 $\mathsf{Shape\;Number}$. The closest rectangle of order 18 is a rectangle of order 18 is a rectangle, requiring subdivision subdivision is a rectangle, C σ the basic rectangle as shown in Fig. 11.18(c), where the chain-code direction σ

 \blacksquare It is advisable to normalize the grid orientation by aligning the chain code grid to $\frac{1}{2}$ the basic rectangle. t is advisable to normalize the grid orientation by aligning <mark>t</mark>h **FIGURE 11.18** c d

 2 Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1 0 0 3 2 2 3 2 2
1 0 3 3 0 1 3 0
0 3 3 0 1 3 0 0
Kundan Kum. 3 0 0 3 2 2 3 2
3 1 0 3 3 0 1 3
1 0 3 3 0 1 3 0
Kundan k Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1 0 3 0 0 3 2 2 3
0 3 1 0 3 3 0 1
3 1 0 3 3 0 1 3
Kundan 0 0 3 0 0 3 2 2
0 0 3 1 0 3 3 0
0 3 1 0 3 3 0 1
Kun

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3 Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3 Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3 Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

b

shape number.

18

18

the resulting shape number usually equals because of the way the grid spac-

the procedure outlined in Section 11.1.2 to obtain the chain code. The shape

Fourier Descriptors

Figure: A digital boundary and its representation as a complex sequence. The point (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

■ Each coordinate pair treat as a complex number

$$
\Bigl(s(k)=x(k)+jy(k)\Bigr)
$$

for $k = 0, 1, 2, \ldots, N - 1$.

The discrete Fourier transform (DFT) of $s(k)$ is

$$
a(u) = \sum_{k=0}^{N-1} s(k)e^{-j2\pi uk/N}
$$

for $u = 0, 1, 2, \ldots, N - 1$

 $a(u)$ are Fourier Descriptors.

Once a boundary is described as a 1-D function, statistical moments (mean, variance, and a few higher-order central moments) can be used to describe it.

$$
\mu_n(z) = \sum_{i=0}^{N-1} (z_i - m)^n p(z_i)
$$

$$
m = \sum_{i=0}^{N-1} z_i p(z_i)
$$

r

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Regional Features/Descriptors

Some simple Descriptors

- The *area of a region* is defined as the number of pixels in the region.
- \blacksquare The *perimeter of a region* is the length of its boundary.
- $\;\;\;\;\;$ Compactness of a region, defined as $(perimeter)^2/area$, and is minimal for a disk-shape region.
- A slightly different descriptor of compactness is the *circularity ratio*, defined as the ratio of the area of a region to the area of a circle (the most compact shape).
- Region descriptors:
	- \Box mean and median of the gray levels,
	- \Box minimum and maximum gray-level values, and
	- \Box number of pixels with above and below the mean.

Region Features

■ There are following region features

- □ Colors, e.g., RGB values, HSV value, L*a*b
- □ Intensity, e.g. Gray Values
- Textures

Further texture is divided into two classes:

- \Box Spatial Domain Features
	- Structural Features, e.g., LBP, Wavelets
	- Statistical Features, e.g., GLCM, Orientation Histogram
- Transformed Domain Features
	- Gabor Filters

 \blacksquare An important approach to region description is to quantify its texture content.

Statistical approaches microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.) Figure: The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical

> *n*th moment of about the mean is z

- Compute the *histogram* of the *area of interest*.
- $\textcolor{black}{\blacksquare}$ The n^{th} *moment* of z about the mean is

$$
\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i) \qquad m = \sum_{i=0}^{L-1} z_i p(z_i)
$$

- The second moment $(n = 2)$ is of particular importance in texture description. It is a measure of gray-level contrast that can be used to establish descriptors of relative smoothness.
- For example, the texture measure, R_i , is 0 for areas of contrast intensity (the variance is 0 here) and approaches 1 for large value of $\sigma^2(z)$

$$
\boxed{R=1-\frac{1}{1+\sigma^2(z)}}
$$

Texture: Statistical approaches

 \blacksquare The third moment

$$
\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)
$$

is a measure of the skewness of the histogram while the fourth moment is a measure of its relatives flatness.

 Some useful additional texture measures bases on histograms include a measure of "*uniformity*", given by

$$
\text{Uniformity} = \sum_{i=0}^{L-1} p^2(z_i)
$$

 \blacksquare Average entropy measure

Entropy =
$$
-\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)
$$

ly in [t](#page-7-0)[h](#page-8-0)[is](#page-9-0)[m](#page-12-0)[e](#page-13-0)[a](#page-14-0)[s](#page-15-0)[u](#page-16-0)[r](#page-17-0)[e](#page-18-0)[.](#page-19-0)[A](#page-21-0)[s](#page-22-0)[e](#page-24-0)[x](#page-25-0)[p](#page-26-0)ected, t[he](#page-27-0)[s](#page-29-0)[a](#page-30-0)[m](#page-31-0)[e](#page-33-0) comments hold [f](#page-34-0)[o](#page-35-0)[r](#page-36-0) [b](#page-41-0)[e](#page-42-0)[c](#page-43-0)[a](#page-44-0)[u](#page-45-0)[s](#page-46-0)[e](#page-47-0)[i](#page-49-0)[t](#page-50-0)

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Texture: Statistical approaches essentially the standard deviation. The standard o ment generally is useful for determining the degree of symmetry of symmetry of $\frac{1}{2}$

Figure: Texture measures for the subimages shown in previous slide

■ The histogram of a digital image with intensity levels in the range $[0, L - 1]$ is a discrete function

$$
h(r_k) = n_k \tag{1}
$$

where, r_k is the kth intensity value and n_k is the number of pixels in the image with intensity r_k

■ Normalized histogram

$$
p(r_k) = \frac{r_k}{MN} \quad \text{for } k = 0, 1, 2, \dots, L - 1.
$$
 (2)

 $p(r_k)$ is an estimate of the probability of occurrence of intensity level r_k in an image.

Compute the histogram of the given image. First find out the number graylevels in the image (how many bit image?).

[Feature Extraction](#page-1-0) Boundary Representation Boundary Descriptors Regional Descriptors [References](#page-51-0) array on the right is matrix **G**. We see that element (1, 1) of **G** is 1, because therm is one of a pixel value in the o[c](#page-24-0)c[ur](#page-26-0)rence in original proportions
thermal process an[d](#page-48-0)a process a process a process a process and process a process a process and process a process a process and process a process a pr ponono

mediately to its right. The other elements of **G** are computed in this manner.

Texture: Gray level co-occurrence matrix (GLCM) Theoreman three occurrence matrix (GLCM) and contain the containment and a pixel value of 6 μ

 \blacksquare Gray Level Co-occurance Matrix: $G_{l,\theta}(i,j)$ where $i = 0, 1, 2, \ldots, L - 1$, $i = 0, 1, 2, \ldots, L - 1$, L is maximum intensity level.

 Above calculation is just for demonstration. For real images, GLCM matrix dimension is $L \times L$, where index varies as $i = 0, 1, 2, ..., L - 1$, $j = 0, 1, 2, ..., L - 1$.

 \blacksquare Maximum probability: Measure of the strongest response of G. The range of value is [0, 1].

Maximum probability $=\max p_{ij}$ $_{i,j}$

Contrast: A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (When G is constant) to $(L-1)^2.$

$$
Contrast = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} (i-j)^2 p_{ij}
$$

Inverse Element Difference Moment: A measure of intensity contrast between a pixel and its neighbor.

$$
\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \frac{p_{ij}}{(i-j)^k} \qquad \text{for } i \neq j
$$

GLCM Features

■ *Uniformity/Energy*: A measure of how intensities are uniformly distributed.

$$
\text{Uniformity} = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij}^{2}
$$

 \blacksquare Homogeneity: Measures the spatial closeness of the distribution of elements in G to the diagonal. The range of values is $[0,1]$, with the maximum being achieved when G is a diagonal matrix.

Homogeneity =
$$
\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \frac{p_{ij}}{1 + |i - j|}
$$

also defined as

Homogeneity =
$$
\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \frac{p_{ij}}{1 + (i - j)^2}
$$

GLCM Features

Entropy: Measures the randomness of the elements of G. The entropy is 0 when all p_{ij} 's are 0 and is maximum when all p_{ij} 's are equal. The maximum value is $2\log_2 L$.

Entropy =
$$
- \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij} \log_2 p_{ij}
$$

Correlation: A measure of how correlated a pixel is to its neighbor over the entire image. Range of values is 1 to -1 .

$$
\begin{aligned}\n\text{Correlation} &= \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \frac{(i - m_r)(j - m_c)p_{ij}}{\sigma_r \sigma_c} \\
m_r &= \sum_{i=0}^{L-1} i \sum_{j=0}^{L-1} p_{ij} \qquad m_r = \sum_{j=0}^{L-1} j \sum_{i=0}^{L-1} p_{ij} \\
\sigma_r^2 &= \sum_{i=0}^{L-1} (i - m_r)^2 \sum_{j=0}^{L-1} p_{ij} \qquad \sigma_c^2 = \sum_{j=0}^{L-1} (j - m_c)^2 \sum_{i=0}^{L-1} p_{ij}\n\end{aligned}
$$

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GLCM feature Visualization

Local Binary Pattern

Basic Local Binary Pattern is governed by

$$
\boxed{b_k = \left\{ \begin{array}{ll} 1 & \text{if} \quad g_k \geqslant g(x) \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{and} \quad \left(\underline{LBP_{ri}(x) = \min \left\{P_j\right\} } \right)}
$$

where P_i is decimal equivalent of binary sequence b_i .

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Local Binary Pattern: Example

Can you compute LBP at the position (?)?

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Local Binary Pattern: Example

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