

Numerical Methods

Lecture 06: Curve Fitting and Interpolation

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- \blacksquare In many scientific and engineering experients, observations of physical quantities are measured and recorded.
- $\;\;\;\;\;$ For example, the strength of many metals depends on the size of the grains. example, the strength of many metals depends on the size of the grains.
- \blacksquare Testing specimens with different grain sizes yields a discrete set of numbers $(d$ average grain diameter, σ_y - yield strength) as

Table 6-1: Strength-grain size data.

 \blacksquare The experimental records are typically referred to as data points. 193 The experimental records are typically referred to as data points.

- Often the data is used for developing, or evaluating, mathematical formulas (equations) that represent the data.
- This is done by curve fitting in which a specific form of an equation is assumed, or pro vided by a guiding theory, and then the parameters of the equation are determined such that the equation best fits the data points.
- Curve fitting can be carried out with many types of functions and with polynomials of various orders.
- Sometimes the data points are used for estimating the expected values between the known points, a procedure called interpolation,
- **For predicting how the data might extend beyond the range over which it was** measured, a procedure called extrapolation.

Curve Fitting

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- Curve fitting is a procedure in which a mathematical formula (equation) is used to best fit a given set of data points. k nown points, a procedure called interpolation, or for predicting how predicting how predicting how predicting how k
- \blacksquare The objective is to find a function that fits the data points overall. This means that the function does not have to give the exact value at any single point, but fits the data well overall.

- \blacksquare A curve that shows the best fit of a power function $(\sigma = C d^m)$ to the data points.
- \blacksquare It can be observed that the curve fits the general trend of the data but does not match any of the data points exactly. For example, $\frac{1}{2}$ shows the data points $\frac{1}{2}$ shows the data points from $\frac{1}{2}$
- \blacksquare Generally, all experimental measurements have built-in errors or uncertainties, and requiring a curve fit to go through every data point is not beneficial.

points have some error, or scatter. Generally, all experimental measure-

Curve Fitting with a linear equation

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1.2 Curve Fitting Fitting References [Background](#page-1-0) Curve-Fitting **Linear-Equation** Nonlinear-Equation [Interpolation](#page-25-0) [Standard](#page-28-0) [Lagrange](#page-30-0) [Newton's](#page-39-0) - [References](#page-51-0) 000000000

Curve Fitting with a linear equation 6.2 Curve Fitting with a Linear Equation 195

 \blacksquare Curve fitting using a linear equation (first degree polynomial) is the process by which an equation of the form:

$$
y = a_1 x + a_0
$$

is used to best fit given data points.

 \blacksquare This is done by determining the constants a1 and a0 that give the smallest error when the data points are substituted in the equation. $\frac{1}{2}$

Measuring How Good Is a Fit

- The fit between given data points and an approximating linear function is determined by first calculating the error, also called the residual, which is the difference between a data point and the value of the approximating function, at each point.
- Subsequently, the residuals are used for calculating a total error for all the points.
- \blacksquare The residual $r;$ at a point, (x_i, y_i) , is the difference between the value y_i of the data point and the value of the function $f(x_i)$ used to approximate the data points:

$$
\boxed{r_i = y_i - f(x_i)}
$$

Measuring How Good Is a Fit

 A criterion that measures how well the approximating function fits the given data can be obtained by calculating a total error E in terms of the residuals.

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$$
E = \sum_{i=1}^{n} r_i = \sum_{i=1}^{n} [y_i - (a_1 x_i + a_0)]
$$

or
$$
E = \sum_{i=1}^{n} |r_i| = \sum_{i=1}^{n} |y_i - (a_1 x_i + a_0)|
$$

or
$$
E = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} [y_i - (a_1 x_i + a_0)]^2
$$

A smaller E in indicates a better fit. This measure can be used to evaluate or compare proposed fits, and last eqation can be used to calculate the coefficients a_1 and a_0 in the linear function.

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Linear Least-Squares Regression

- An experiment produces a set of data points $(x_1, y_1), \ldots, (x_n, y_n)$, where are abscissas $\{x_k\}$ are distint.
- One goal of numerical methods is to determine a formula $y = f(x)$ that relates these variables.

$$
y = f(x) = a_1 x + a_0
$$

- **Linear least-squares regression is a procedure in which the coefficients** a_1 and a_0 of a linear function $y = a_1x + a_0$ are determined such that the function has the best fit to a given set of data points.
- The best fit is defined as the smallest possible total error that is calculated by adding the squares of the residuals

$$
E = \sum_{i=1}^{n} [y_i - (a_1 x_i + a_0)]^2
$$

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 \blacksquare Take the partial derivative of of above equation, we get ting them equal to zero gives:

$$
\frac{\partial E}{\partial a_0} = -2 \sum_{i=1}^n (y_i - a_1 x_i - a_0) = 0
$$

$$
\frac{\partial E}{\partial a_1} = -2 \sum_{i=1}^n (y_i - a_1 x_i - a_0) x_i = 0
$$

at the values of a 1 and a0 where the partial derivatives of E with respect

equations are a system of two linear equations for the unknows \overline{a}_1 a_0 , and can be rewritten in the form as Above two equations are a system of two linear equations for the unknows a_1 and
 a_0 , and can be rewritten in the form as
 $na_0 + \left(\sum_{i=1}^n x_i\right) a_1 = \sum_{i=1}^n y_i$
 $\left(\sum_{i=1}^n x_i\right) a_0 + \left(\sum_{i=1}^n x_i^2\right) a_1 = \sum_{i=1}^n x_i$

$$
na_0 + \left(\sum_{i=1}^n x_i\right) a_1 = \sum_{i=1}^n y_i
$$

$$
\left(\sum_{i=1}^n x_i\right) a_0 + \left(\sum_{i=1}^n x_i^2\right) a_1 = \sum_{i=1}^n x_i y_i
$$

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Linear Least-Squares Regression $\frac{1}{2}$ Chapter 6 Curve Fitting and Interpolation $\frac{1}{2}$

 \blacksquare Solution can be written as

 \blacksquare The values of a_1 and a_0 in the equation $y = a_1x + a_0$ that has the best fit to n data points (x_i, y_i)

$$
a_1 = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2}
$$

$$
a_0 = \frac{S_{xx}S_y - S_{xy}S_x}{nS_{xx} - (S_x)^2}
$$

where,

$$
S_x = \sum_{i=1}^n x_i, \quad S_y = \sum_{i=1}^n y_i, \quad S_{xy} = \sum_{i=1}^n x_i y_i, \quad S_{xx} = \sum_{i=1}^n x_i^2
$$

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Curve Fitting with Nonlinear Equation

Curve Fitting with Nonlinear Equation and engineering show that the relationship biningar Equation that are being considered is not linear.

- \blacksquare Many situations in science and engineering show that the relationship between the quantities that are being considered is not linear.
- \blacksquare For example, the data points meansured in RC circuit.

 \blacksquare It is obvious from the plot that curve fitting the data points with a nonlinear function gives a much better fit than curve fitting with a linear function.

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	- \blacksquare There are many kinds of nonlinear functions which can be used with linear-squares regression method to determine the coefficients that gives the best fit. which the linear squares regression method can be used for three squares regression and the use of determined be used for determined by y : by the basic function of the second control of the control of the function of the control of the control of $\frac{1}{2}$ or $\frac{1}{2}$ are the coefficients that gives the best fit
	- \blacksquare Examples of nonlinear functions used for curve fitting in the present section are:

$$
y = bx^{m}
$$
 (power function)
\n
$$
y = be^{mx} \text{ or } y = b10^{mx}
$$
 (exponential function)
\n
$$
y = \frac{1}{mx + b}
$$
 (reciprocal function)

- \blacksquare In order to be able to use linear regression, the form of a nonlinear equation of two variables is changed such that the new form is linear with terms that contain the variables is enarigents. t erms that contain the original variables. For example, the power function $\mathcal{F}_{\mathcal{A}}$
- \blacksquare For example, the power function $y = bx^m$ can be put into linear form by taking the natural logarithm (ln) of both sides:

$$
\ln(y) = \ln(bx^m) = m\ln(x) + \ln(b)
$$

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 \blacksquare This equation is linear for $ln(y)$ in terms $ln(x)$. The equation is in the form $Y = a1X + a0$ where $Y = ln(y)$, $a1 = m, X = ln(x)$, and $a0 = ln(b)$:

$$
\ln(y) = m \ln(x) + \ln(b)
$$
\n
$$
Y = a_1 X + a_0
$$

- **This means that linear least-squares regression can be used for curve fitting an** equation of the form $y = bx^m$ to a set of data points $x_i, y_i.$
- \blacksquare Once a_1 and a_0 are known, the constants b and m in the exponential equation are and α is the substitution of α ; and ln(y;) for α ; and ln(x); α and ln(x); α calculated by:

$$
m = a_1 \qquad \text{and} \qquad b = e^{a_0}
$$

Transforming nonlinear equations to linear form and the constant of the control

[Background](#page-1-0) Curve-Fitting Linear-Equation **Nonlinear-Equation** [Interpolation](#page-25-0) [Standard](#page-28-0) [Lagrange](#page-30-0) [Newton's](#page-39-0) - [References](#page-51-0) How to choose an appropriate nonlinear function for curve fitting

Curve fitting with quadratic and higher order polynomials

Curve fitting with quadratic and higher order polynomials

Background:

Polynomials are functions that have the form:

$$
f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0
$$

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The coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers, and n, which is a nonnegative integer, is the degree, or order, of the polynomial.

- \blacksquare A plot of the polynomial is a curve. A first-order polynomial is a linear function, and its plot is a straight line. Higher-order polynomials are nonlinear functions, and their plots are curves.
- A quadratic (second-order) poly nomial is a curve that is either concave up or down (parabola).
- A third-order polynomial has an inflection point such that the curve can be con cave up (or down) in one region, and concave down (or up) in another.

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Curve fitting with quadratic and higher order polynomials

- A given set of n data points can be curve-fit with polynomials of different order up to an order of $(n-1)$.
- The coefficients of a polynomial can be determined such that the polynomial best fits the data by minimizing the error in a least squares sense.
- For n points the polynomial that passes through all of the points is one of order $(n-1)$. Even though the high-order polynomial gives the exact values at all the data points, it cannot be used reliably for interpolation or extrapolation.

- Polynomial regression is a procedure for determining the coefficients of a polynomial of a second degree, or higher, such that the polynomial best fits (minimizing the total error) a given set of data points.
- If the polynomial, of order m, that is used for curve fitting is:

$$
f(x) = a_m x^m + a_{m-1} x^{m-1} + \ldots + a_1 x + a_0
$$

■ Then, for a given set of n data points $\{(x_i, y_i)\}_{i=1}^n$ $(m$ is smaller than $n-1)$, the total error is given by:

$$
E = \sum_{i=1}^{n} \left[y_i - (a_m x_i^m + a_{m-1} x_i^{m-1} + \dots + a_1 x_i + a_0) \right]^2
$$

The function E has a minimum at the values of a_0 through a_m where the partial derivatives of E with respect to each of the variables is equal to zero.

[Background](#page-1-0) Curve-Fitting Linear-Equation **Nonlinear-Equation** Interpolation Standard Lagrange [Newton's](#page-39-0) - [References](#page-51-0) Polynomial regression through am). The function E [has a minim](#page-25-0)u[m at the](#page-28-0) v[alues of](#page-30-0) a0 through [a](#page-41-0)[m](#page-10-0)w[h](#page-34-0)er[e](#page-39-0) the [p](#page-14-0)[ar](#page-16-0)[t](#page-17-0)[i](#page-18-0)a[l](#page-20-0) [d](#page-21-0)[e](#page-31-0)[r](#page-23-0)[iv](#page-24-0)ati[v](#page-40-0)e[s](#page-27-0) of E wit[h](#page-28-0) [re](#page-29-0)spect t[o](#page-30-0) the various to ea[c](#page-33-0)h [o](#page-36-0)[f](#page-37-0) [t](#page-38-0)he va[ri](#page-42-0)[-](#page-43-0) $\mathbf s$ ion. Taking the partial derivatives of $\mathbf s$ in Eq. (6.21). The partial derivatives of $\mathbf s$

 \blacksquare For the simplicity, let us consider the case of $m=2$ (quadratic polynomial) ϵ , is as consider the case of $m = 2$ (quadratic polynomial)

$$
E = \sum_{i=1}^{n} [y_i - (a_2 x_i^2 + a_1 x_i + a_0)]^2
$$

and setting them to zero gives a set of m \sim 1 linear equations for the \sim

 \blacksquare Taking the partial derivatives with respect to a0, a1, and a2, and setting them equal to zero gives: them equal to zero gives:

$$
\frac{\partial E}{\partial a_0} = -2 \sum_{i=1}^n (y_i - a_2 x_i^2 - a_1 x_i - a_0) = 0
$$

$$
\frac{\partial E}{\partial a_1} = -2 \sum_{i=1}^n (y_i - a_2 x_i^2 - a_1 x_i - a_0) x_i = 0
$$

$$
\frac{\partial E}{\partial a_2} = -2 \sum_{i=1}^n (y_i - a_2 x_i^2 - a_1 x_i - a_0) x_i^2 = 0
$$

for the unknowns a0, a1, and a2, which can be rewritten in the form:

$$
na_0 + \left(\sum_{i=1}^n x_i\right)a_1 + \left(\sum_{i=1}^n x_i^2\right)a_2 = \sum_{i=1}^n y_i
$$

$$
\left(\sum_{i=1}^n x_i\right)a_0 + \left(\sum_{i=1}^n x_i^2\right)a_1 + \left(\sum_{i=1}^n x_i^3\right)a_2 = \sum_{i=1}^n x_i y_i
$$

$$
\left(\sum_{i=1}^n x_i^2\right)a_0 + \left(\sum_{i=1}^n x_i^3\right)a_1 + \left(\sum_{i=1}^n x_i^4\right)a_2 = \sum_{i=1}^n x_i^2 y_i
$$

- of the coefficients a0, a1, and a2 of the polynomial \blacksquare The solution of the system of equations gives the values of the coefficients $a0,~a1,$ and $a2$ of the polynomial $y = a_2x_i^2 + a_1x_i + a_0$ that best fits then data points same way. For an multiplier polynomial, \mathcal{L} $\{(x_i, y_i)\}_{i=1}^n$.
- The coefficients for higher-order polynomials are derived in the same way.

the figure, and their values are given below. Determine the '2' ⁰� 30 ⁰

Interpolation

- Interpolation is a procedure for estimating a value between known val ues of data points.
- \blacksquare It is done by first determining a polynomial that gives the exact value at the data points, and then using the polynomial for calculating values between the points.

gives the exact value at the data points, and then using the polynomial \blacksquare When a small number of points is involved, a single polynomial might be sufficient for interpola tion over the whole domain of the large number of points are involved, different polynomials are used in data points.

equations. Curve fitting can be calculated out with α

through the points precisely and gives a good estimate of values

 \blacksquare Often, however, when a large number of points are involved, different polynomials are used in the intervals between the points in a process that is called spline inter polation.

Interpolation

- As discussed in curve fitting, for any number of points n there is a polynomial of order $n - 1$ that passes through all of the points.
- First, second, third, and fourth-order polynomials connect two, three, four, and five points, respectively.

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Interpolation using a single polynomial

- \blacksquare Interpolation with a single polynomial gives good results for a small number of points.
- \blacksquare For a large number of points the order of the polynomial is high, and although the polynomial passes through all the points, it might deviate significantly between the points. 4 4

- Consequently, interpolation with a single polynomial might not be appropriate for a large number of points.
- For a large number of points, better interpolation can be done by using piecewise (spline) interpolation in which different lower-order polynomials are used for interpolation between different points of the same set of data points.
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	- For a given set of n points, only one (unique) polynomial of order m $(m = n 1)$ passes exactly through all of the points.
	- The polynomial, however, can be written in different mathematical forms.
	- Three forms of polynomials are
		- □ Standard.
		- Lagrange, and
		- Newton's
	- Standard form of an mth-order polynomial is:

$$
f(x) = a_m x^m + a_{m-1} x^{m-1} + \ldots + a_1 x + a_0
$$

- **The coefficients in this form are determined by solving a system of** $m + 1$ linear equations.
- The equations are obtained by writing the polynomial explicitly for each point (substituting each point in the polynomial). (Refer to curve fitting)

Lagrange Interplating Polynomials

Lagrange Interplating Polynomials

- Lagrange interpolating polynomials are a particular form of polynomi als that can be written to fit a given set of data points by using the val ues at the points.
- **The polynomials can be written right away and do not require any preliminary** calculations for determining coefficients.
- **Lagrange polynomials**
	- First-order Lagrange polynomial
	- Second-order Lagrange polynomial
	- □ General form of an $n-1$ order Lagrange polynomial

First order Lagrange polynomial and Using a Single Polynomial 213 Interpolation Using a Single Polynomial 213

 \blacksquare For two points, (x_1,y_1) , and (x_2,y_2) , the first-order Lagrange polynomial that passes through the points has the form: m : m :

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$$
f(x) = y = a_1(x - x_2) + a_2(x - x_1)
$$

Substitute the two point in the above equation

 $\mathcal{L}_{\mathcal{A}}$ is the single Polynomial 213 Interpolation Using a Single Polynomial 213 Interpolation $\mathcal{L}_{\mathcal{A}}$

 $S_{\rm eff}$ two points in Eq. (6.37) gives: \sim

$$
y_1 = a_1(x_1 - x_2) + a_2(x_1 - x_1)
$$
 or $a_1 = \frac{y_1}{(x_1 - x_2)}$
 $y_2 = a_1(x_2 - x_2) + a_2(x_2 - x_1)$ or $a_2 = \frac{y_2}{(x_2 - x_1)}$

Interpolation Standard Lagrange Newton's References

Substitute a_1 and a_2 back in Eq. (6.40), the value of the polynomial is y1, and if x = x2 is substidustitute u_1 and u_2 back

$$
f(x) = \frac{(x - x_2)}{(x_1 - x_2)} y_1 + \frac{(x - x_1)}{(x_2 - x_1)} y_2
$$

$$
f(x) = \frac{(y_2 - y_1)}{(x_2 - x_1)} x_1 + \frac{x_2 y_1 - x_1 y_2}{(x_2 - x_1)}
$$

between the points gives an interpolated value of α

tuted, the value of the polynomial is Yi· Substituting a value of x

Lagrange polynomial.

Second-order Lagrange polynomial and (x1), (x2, y2), and (x3, y3), and (x3, y2), the second-order \sim $\mathcal{L}_{\mathbf{S}}$ three points, (x2, y2), $\mathcal{L}_{\mathbf{S}}$, $\mathcal{L}_{\mathbf{S}}$, $\mathcal{L}_{\mathbf{S}}$, $\mathcal{L}_{\mathbf{S}}$

 \blacksquare For three points, (x_1,y_1) , (x_2,y_2) , and (x_3,y_3) , the second-order Lagrange polynomial that passes through the points has the form: form: Lagrange polynomial that passes through the points (Fig. 6-13) has the ${\rm ge}$

[L](#page-18-0)[a](#page-19-0)[g](#page-20-0)[r](#page-21-0)[a](#page-22-0)nge polynomial.

<mark>lonlinear Eq</mark>i

$$
f(x) = y = a_1(x - x_2)(x - x_3) + a_2(x - x_1)(x - x_3) + a_3(x - x_1)(x - x_2)
$$

 \blacksquare Once the coefficients are determined such that the polynomial passes through the three points, the polynomial (quadratic form) is: $\overline{}$

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Standard Lagrange [\(x](#page-23-0)[2](#page-24-0)-[x](#page-25-0)[1](#page-26-0)[\)](#page-27-0) (x[2](#page-28-0)[-](#page-29-0)x1) (6.41)

(x2[-x1\)](#page-25-0) ([x2-x1](#page-28-0)) (6.41)

$$
f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} y_3
$$

(Home work to derive the standard form) $\frac{1}{2}$ $\mathcal{F}_{\mathbf{r}}$ \blacksquare Above equation can also be rewritten in the standard form $f(x) = a_2x^2 + a_1x + a_0.$ $a_0.$

$$
f(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} y_2
$$

$$
f(x)
$$

$$
f(x)
$$

Example 2152

Once the coefficients are determined such that the polynomial passes

Newton's References
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[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation Interpolation Standard **Lagrange** [Newton's](#page-39-0) [References](#page-51-0) General form of Lagrange polynomial [f](#page-6-0)or the momentum of the mail of the standard the distringuious ...
France condition the condition of the condition of the conditional conditional conditional conditional conditi
Section the conditional conditional conditio [x](#page-8-0)[=](#page-11-0) [Y1+](#page-25-0) [Y](#page-39-0)[z](#page-40-0)[-+](#page-41-0)(x1-x[2\)](#page-12-0)[\(](#page-13-0)[x](#page-14-0)[1](#page-15-0)[-](#page-16-0)[x](#page-18-0)[3](#page-19-0)[\)](#page-20-0) [..](#page-21-0)[.](#page-22-0) [\(](#page-23-0)[x](#page-24-0)1-[x](#page-25-0)[n](#page-26-0)[\)](#page-27-0) [\(x](#page-25-0)2-[x](#page-28-0)[1\)](#page-29-0)[\(x2-](#page-28-0)x[3\)](#page-30-0) [..](#page-31-0)[.](#page-32-0) [\(](#page-33-0)[x](#page-34-0)[2](#page-35-0)[-](#page-36-0)[xn](#page-38-0))

2 C ¹ C ¹ ² C ¹ ² C ¹ ² Curve Fig. 2014 **and Interpolation and Interpolation and Interpretation and Interpretation and Interpretation and Interpretation and Interpretation and Interpretation a** the gene points (x_1, y_1) , (x_2, y_2) , ..., (x_n, y_n) is:

$$
f(x) = \frac{(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)} y_1 + \frac{(x-x_1)(x-x_3)\dots(x-x_n)}{(x_2-x_1)(x_2-x_3)\dots(x_2-x_n)} y_2 + \dots + \frac{(x-x_1)(x-x_2)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_1)(x_i-x_2)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} y_i + \dots + \frac{(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} y_n
$$

$$
f(x) = \sum_{i=1}^n y_i L_i(x) = \sum_{i=1}^n y_i \prod_{\substack{j=1 \ j \neq i}}^n \frac{(x-x_j)}{(x_i-x_j)}
$$

where
$$
L_i(x) = \prod_{\substack{j=1 \ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}
$$
 are called the Lagrange functions

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation [Interpolation](#page-25-0) [Standard](#page-28-0) **[Lagrange](#page-30-0)** [Newton's](#page-39-0) [References](#page-51-0) 000000000 00000000000 Conclusive remarks

- The spacing between the data points does not have to be equal.
- \blacksquare For a given set of points, the whole expression of the interpolation polynomial has to be calculated for every value of x . In other words, the interpolation calculations for each value of x are independent of others.
- Different from other forms where once the coefficients of the polynomial are determined, they can be used for calculating different values of x .
- If an interpolated value is calculated for a given set of data points, and then the data set is enlarged to include additional points, all the terms of the Lagrange polynomial have to be calculated again.
- As discussed in next topic, this is different from Newton's polynomials where only the new terms have to be calculated if more data points are added.

Newton's Interpolating Polynomials

Newton's Interpolating Polynomials

- \blacksquare Newton's interpolating polynomials are a popular means of exactly fitting a given set of data points.
- The general form of an $n-1$ order Newton's polynomial that passes through n points is:

 $f(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2) + \ldots + a_n(x-x_1)(x-x_2) \ldots (x-x_{n-1})$

- The special feature of this form of the polynomial is that the coeffi cients a1 through an can be determined using a simple mathematical procedure.
- \blacksquare (Determination of the coefficients does not require a solu tion of a system of n equations.)
- Once the coefficients are known, the polynomial can be used for calculating an interpolated value at any x .

Newton's Interpolating Polynomials

- Newton's interpolating polynomials have additional desirable features that make them a popular choice.
- The data points do not have to be in descending or ascending order, or in any order.
- Moreover, after the n coefficients of an $n-1$ order Newton's interpolating polynomial are determined for n given points, more points can be added to the data set and only the new additional coefficients have to be determined.

First-order Newton's polynomial 6.5 Interpolation Using a Single Polynomial 217 $\frac{1}{200}$ $\frac{1}{200}$ $\frac{1}{200}$ [,](#page-41-0) $\frac{1}{200}$ $\frac{1}{200}$ $\frac{1}{200}$, $\frac{1}{200}$, the first-[o](#page-26-0)r[d](#page-35-0)e[r](#page-52-0) $\frac{1}{200}$, [t](#page-29-0)h[e](#page-45-0) [fi](#page-47-0)r[st](#page-49-0)[-o](#page-50-0)r[de](#page-51-0)r $\frac{1}{200}$

■ For two given points, (x_1,y_1) and (x_2,y_2) , the first-order Newton's polynomial has the form: $p = p$ y en noints (x, y) $A = \begin{pmatrix} \alpha & 1 & 0 \\ 0 & \alpha & 1 \end{pmatrix}$

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation Interpolation Standard Lagrange **Newton's** References

6.5 In[terpolation U](#page-5-0)sin[g a Single Polyn](#page-12-0)o[mial](#page-25-0) 217

Interpolation

$$
f(x) = a_1 + a_2(x - x_1)
$$

It is an equation of a straight line that passes through the points. f(x) ---- B

considering the similar triangles The coefficients a_1 and a_2 can be calculated by ϵ

As shown in Fig. 6-14, it is an equation of a straight line that passes through the points. The coefficients a1 and a2 can be calculated by considering the similar triangles in Fig. 6-14. DE =AB or CE CB' f(x)-Y1 ⁼ Y2-Yi X-Xl Xz-X1 (6.53) The coefficients a1 and a2 can be calculated by DE =AB or CE CB' f(x)-Y1 ⁼ Y2-Yi Solving Eq. (6.48) for f(x) gives: X-Xl Xz-X1 f(x) = Y1 + Y z -y1(x -xi) Xz-XI Solving Eq. (6.48) for f(x) gives: X-Xl Xz-X1 f(x) = Y1 + Y z -y1(x -xi) Xz-XI Comparing Eq. (6.49) with Eq. (6.47) gives the values of the coefficients a1 and a2 in terms of the coordinates of the points: f(x) = Y1 + Y z -y1(x -xi) Xz-XI Comparing Eq. (6.49) with Eq. (6.47) gives the values of the coefficients a1 and a2 in terms of the coordinates of the points: a1 = y1 , and Y2-Y1 az = Xz ---XI

 $\overline{\text{S}}$. The coefficient $\overline{\text{S}}$ is the class of the line thet compatible $x^2 + y^2 = 1$ the two-points. $\frac{1}{2}$ X_2 X_3 \Box The coefficient a_2 is the slope of the line that connects the two points. an \mathbf{r} and \mathbf{r} , and \mathbf{r} , and \mathbf{r} , and \mathbf{r} , and \mathbf{r}

Second-order Newton's polynomial

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Second-order Newton's polynomial [N](#page-16-0)[o](#page-18-0)[t](#page-19-0)[i](#page-20-0)[c](#page-21-0)e [th](#page-24-0)a[t](#page-25-0) [th](#page-26-0)e coe[ffic](#page-28-0)[ie](#page-29-0)nt a[2](#page-30-0) [i](#page-31-0)[s](#page-32-0)[t](#page-34-0)[h](#page-35-0)[e](#page-36-0)[s](#page-38-0)lo[p](#page-39-0)[e](#page-40-0)[o](#page-43-0)[f](#page-44-0)[t](#page-46-0)[h](#page-47-0)[e](#page-48-0)[li](#page-50-0)n[e](#page-51-0) [t](#page-52-0)hat connects the ϵ points. As shown in Chapter 8, a2 is the two-point for ϵ

 \blacksquare For three given points, (x_1,y_1) , (x_2,y_2) , and (x_3,y_3) , the second-order Newton's polynomial has the form:

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation [Interpolation](#page-25-0) Standard Lagrange **Newton's** [References](#page-51-0)

$$
f(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)
$$

It is an equation of a parabola that passes through the three points.

Figure 6-15: Second-order

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 \blacksquare The coefficients $a_1, a_2,$ and a_3 can be determined by substituting the three points in above equation. $f(x) = \frac{1}{2}x^2 + \frac{1}{2}$

 $\frac{X}{2}$

- Substituting $x = x_1$ and $f(x1) = y1$ gives: $a_1 = y_1$ $t_{\rm r}$ through the three points. The coefficients and a α = Jubstituting $x = x_1$ and $f(x_1) = y_1$ gives. $a_1 - y_1$.
- **9** Substituting the second point, $x = x_2$ and $f(x_2) = y_2$, (and $a_1 = y_1$) in above eq. gives: ■ The coefficients a_1 , a_2 , and a_3 can be determined
by substituting the three points in above equation.
■ Substituting $x = x_1$ and $f(x_1) = y_1$ gives: $a_1 = y_1$
■ Substituting the second point, $x = x_2$ and
 $f(x_2)$

[c](#page-14-0)ients [a](#page-22-0)[1](#page-23-0) and [a2](#page-27-0) in term[s of the](#page-28-0) c[oordina](#page-30-0)tes [of the p](#page-39-0)oints:

$$
y_2 = y_1 + a_2(x_2 - x_1)
$$
 or $a_2 = \frac{y_2 - y_1}{x_2 - x_1}$

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation [Interpolation](#page-25-0) [Standard](#page-28-0) [Lagrange](#page-30-0) **[Newton's](#page-39-0)** [References](#page-51-0) 000000000 Second-order Newton's polynomial

Substituting the third point, $x = x_3$ and $f(x_3) = y_3$ (as well as $a_1 = y_1$ and $a_2 = \frac{y_2 - y_1}{y_1}$ in $f(x)$ that gives: $a_2 = \frac{y_2-y_1}{x_2-x_1}$ ■ Substituting the third point, $x = x_3$ and $f(x_3) = y_3$ (as well as $a_1 = y_1$ and $a_2 = \frac{y_2 - y_1}{x_2 - x_1}$) in $f(x)$ that gives: Newton's polynomial. $S_{\rm 3D}$ and $S_{\rm 3D}$ third point, x_{3} and y_{3} and y_{3} (x_{3}) z_{3} (x_{3}) z_{3} $\ddot{}$ is $\ddot{}$ if $\ddot{}$ is $\ddot{}$ if $\ddot{}$ is $\ddot{}$

$$
y_3 = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)
$$

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Above equation can be solved for a_3 and rearranged to give (after some algebra):

$$
\frac{(-y_1}{-x_1}(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)
$$

or a_3 and rearranged to give (after some algebra):

$$
\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}
$$

$$
\frac{y_3 - y_2}{(x_3 - x_1)}
$$
the same in the first-order and sec ond-order
by equation, the equation

 \blacksquare The coefficients a1, and a2 are the same in the first-order and sec ond-order polynomials. This means that if two points are given and a first-order Newton's polynomial is fit to pass through those points, and then a third point is added, the polynomial can be changed to be of sec ond-order and pass through the three points by only determining the value of one additional coefficient. value of one additional coefficient. The coefficient of one additional coefficient. bass through those points, and then a third
hanged to be of sec ond-order and pass three mining the value of one additional coefficient
Kundan Kumar 41/52 Kundan Kumar Pattern Classification

Third-order Newton's polynomial Third-order Newton's polynomial Third-order Newton's polynomial $\mathcal{F}_{\mathcal{A}}$ four given points, ($\mathcal{A}_{\mathcal{A}}$

 \blacksquare For four given points, (x_1,y_1) , (x_2,y_2) , (x_3,y_3) and (x_4,y_4) , the third-order Newton's polynomial that passes through the four points has the form: $\frac{f(x)}{1-x}$, (x1, y1), (x1), (x2, Jii), (x2, omis, (u_1, y_1) , (u_2, y_2) , (u_3, y_3) and (u_4, y_4) , the finite-order $\sum_{i=1}^{\infty}$ order $\sum_{i=1}^{\infty}$ ($\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ and $\sum_{i=1}^{\infty}$ the third-order n_{ol}

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Curve Fitting Linear Equation Monlinear Equation Interpolation

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation Interpolation Standard Lagrange **[Newton's](#page-39-0)** [References](#page-51-0)

$$
f(x) = y = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2) + a_4(x-x_1)(x-x_2)(x-x_3)
$$

), (x3) and (x3) and (x3) and (x3), the third-control \mathcal{A}

 \blacksquare The formulas for the coefficients a1, a2, and a3 are the same as for the second order polynomial. The formula for the coefficient a4 can be obtained by substituting (x_4, y_4) , in Eq and solving for a_4 , which gives:

gives: (y4-y3 Y3-Y2) (y3-Y2 Y2-Y1) X4 -X3 X3 -X2 X3 -X2 X2 -XI (x4-x2) (x3-x1) a4 = [--](#page-25-0)[-----------](#page-40-0) (x4 -X1) (y4-y3 Y3-Y2) (y3-Y2 Y2-Y1) X4 -X3 X3 -X2 X3 -X2 X2 -XI (x4-x2) (x3-x1) a4 = [--](#page-12-0)----------- (x4 -X1)

 A careful examination of the equation of the equations \mathcal{C}

A general form of Newton's polynomial and its coefficients $A_{\rm eff}$ is the coefficients for the equation for the coefficients and $B_{\rm eff}$

- \blacksquare There is common pattern in all equations that can be clarified by defining so called divided differences. F two points, $\frac{1}{2}$, $\frac{1}{2$
- \blacksquare For two points, (x_1,y_1) , and (x_2,y_2) , the first divided difference, written as $f[x2,xi]$, is defined as the slope of the line connecting the two points:

$$
f[x_2, x_1] = \frac{y_2 - y_1}{x_2 - x_1} = a_2
$$

 \blacksquare The first divided difference is equal to the coefficient $a_2.$

A general form of Newton's polynomial and its coefficients

For three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) the second divided difference, written as $f[x_3,x_2,x_1]$, is defined as the difference between the first divided differences of points (x_3, y_3) , and (x_2, y_2) , and points (x_2, y_2) , and (x_1, y_1) divided by $(x_3 - x_1)$:

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation [Interpolation](#page-25-0) [Standard](#page-28-0) [Lagrange](#page-30-0) **[Newton's](#page-39-0)** [References](#page-51-0)

$$
y_3), \text{ and } (x_2, y_2), \text{ and points } (x_2, y_2), \text{ and } (x_1, y_1) \text{ divided by}
$$
\n
$$
f[x_3, x_2, x_1] = \frac{f[x_3, x_2] - f[x_2, x_1]}{x_3 - x_1} = \frac{\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}}{(x_3 - x_1)} = a_3
$$
\ndivided difference is thus equal to the coefficient a_3

 \blacksquare The second divided difference is thus equal to the coefficient a_3

A general form of Newton's polynomial and its coefficients

 \blacksquare For four points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and (x_4, y_4) the third divided difference, written as $f[x_4, x_3, x_2, x_1]$, is defined as the differ ence between the second divided differences of points (x_2,y_2) , (x_3,y_3) and (x_4,y_4) , and points (x_1,y_1) , (x_2,y_2) , and (x_3,y_3) divided by $(x_4 - x_1)$: $\mathcal{F}_{\mathcal{A}}(x,y)$, $\mathcal{F}_{\mathcal{A}}(x,y)$ the third (x4), $\mathcal{F}_{\mathcal{A}}(x,y)$ the third (x4) the third (

[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation Interpolation Standard [Lagrange](#page-30-0) **Newton's** [References](#page-51-0)

f [x4, x3, x2] -f [x3, x2, xi] f[x4,X3,x2,xil = -- ----....;;;;_ X4-X1 f [x4, X3] -f [x3, X2] f [x3, X2] -f [x2, xi] (X4 -X1) (6.59) Y4-Y3 Y3-Yi Y3-Y2 Yi-Yi X4 -X3 X3 -X2 X3 -X2 X2 -X1 [--](#page-12-0)[----------](#page-28-0) (x4 -x2) (x3 -x1) ⁼a4 (X4-X1)

- \blacksquare The third divided difference is thus equal to the coefficient $a_4.$
- If more data points are given, the procedure for calculating higher dif ferences continues in the same manner.

Newton's
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[Background](#page-1-0) Curve-Fitting Linear-Equation Nonlinear-Equation [Interpolation](#page-25-0) [Standard](#page-28-0) [Lagrange](#page-30-0) **[Newton's](#page-39-0)** [References](#page-51-0)

A general form of Newton's polynomial and its coefficients

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A general form of Newton's polynomial and its coefficients

■ In general, when n data points are given, the procedure starts by calculating $\left(n-1\right)$ first divided differ ences. Then, $\left(n-2\right)$ second divided differences are calculated from the first divided differences. This is followed by calculating $(n-3)$ third divided differences from the second divided differences. The process ends when one nth divided difference is calculated from two $\left(n-1\right)$ divided differences to give the coefficient a_n . In general terms, for n given data points,

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ferences continues in the same manner. In the same manner of the same manner. In the same manner, when the poi are [g](#page-5-0)[i](#page-6-0)[v](#page-7-0)[e](#page-9-0)[n](#page-10-0)[,](#page-11-0) the p[ro](#page-12-0)[c](#page-14-0)[e](#page-15-0)[d](#page-16-0)[u](#page-18-0)[r](#page-19-0)[e](#page-20-0)[s](#page-22-0)[t](#page-23-0)[ar](#page-24-0)ts [b](#page-25-0)[y](#page-27-0) calcula[ti](#page-28-0)[n](#page-29-0)g (n [-](#page-30-0) [1](#page-31-0)[\)](#page-32-0)[fi](#page-34-0)[r](#page-35-0)[s](#page-36-0)[t](#page-37-0) [d](#page-38-0)i[v](#page-39-0)[i](#page-40-0)[d](#page-41-0)[e](#page-42-0)[d](#page-44-0)[d](#page-46-0)[i](#page-47-0)[ff](#page-48-0)[er](#page-50-0)-

 \blacksquare In general terms, for n given data points, (x_1,y_1) , (x_2,y_2) , \dots , (x_n,y_n) , the first divided differences between two points (x_i, y_i) , and (x_j, y_j) are given by:

$$
f[x_j, x_i] = \frac{y_j - y_i}{x_j - x_i}
$$

A general form of Newton's polynomial and its coefficients Figure 6-16: Table of divided differences for five data points. CurveFitting Einear

Nonlinear Equation
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[f](#page-11-0) [X5, X4] /" f [X5, X4, X3]

 \blacksquare The kth divided difference for second and higher divided differences up to the the fight annual difference for second and inglue α .
 $(n-1)$ difference is given by:

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Interpolation Standard Lagrange

$$
f[x_k, x_{k-1}, ..., x_2, x_1] = \frac{f[x_k, x_{k-1}, ..., x_3, x_2] - f[x_{k-1}, x_{k-2}, ..., x_2, x_1]}{x_k - x_1}
$$

With these definitions, the $(n-1)$ order Newton's polynomial, Eq. $(6, 46)$ is given by: $\mathcal{L}(\mathcal{L})$ order $\mathcal{L}(\mathcal{L})$ order Newton's polynomial, $\mathcal{L}(\mathcal{L})$

With these definitions, the (n - 1) order Newton's polynomial, Eq. (6. 46) is given by:

\n
$$
f(x) = y = y_1 + f[x_2, x_1](x - x_1) + f[x_3, x_2, x_1](x - x_1)(x - x_2) + \ldots + f[x_n, x_{n-1}, \ldots, x_2, x_1](x - x_1)(x - x_2) \ldots (x - x_{n-1})
$$
\nand

\n
$$
g_1 = \begin{bmatrix}\na_1 \\
a_2 \\
a_3\n\end{bmatrix}
$$
\nExample 1. The equation is given by the formula for the following equations:

\n
$$
g(x) = \begin{bmatrix}\na_1 \\
a_2 \\
a_3\n\end{bmatrix}
$$
\nExample 2. The equation is given by the formula for the equation $f(x) = y_1 + f[x_2, x_1](x - x_1) + f[x_3, x_2, x_1](x - x_1)(x - x_2) + \ldots + f[x_n, x_{n-1}, \ldots, x_2, x_1](x - x_1)(x - x_2) \ldots (x - x_{n-1})$ \nand

\n
$$
g(x) = \begin{bmatrix}\na_1 \\
a_2 \\
a_3\n\end{bmatrix}
$$
\nExample 3. The equation is given by the formula for the equation $f(x) = y_1 + f[x_2, x_1](x - x_1) + f[x_3, x_2, x_1](x - x_1)(x - x_2) + \ldots + f[x_n, x_{n-1}, \ldots, x_2, x_1](x - x_1)(x - x_2) \ldots (x - x_{n-1})$

Curve Fitting Linear Equ
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Linear Equation

Backgrou[n](#page-1-0)d
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 \sim The spacing between the data points do not have to be the same. The same \sim

Newton'sR[e](#page-51-0)ferences
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Notes about Newton's polynomials

- The spacings between the data points do not have to be the same.
- For a given set of n points, once the coefficients a_1 through a_n are determined, they can be used for interpolation at any point between the data points.
- After the coefficients a_1 through a_n are determined (for a given set of n points), additional data points can be added (they do not have to be in order), and only the additional coefficients have to be determined.

Piecewise (Spline Interpolation)

[1] Amos Gilat and Vish Subramaniam. Numerical Methods for Engineers and Scientists: An Introduction with Applications using MATLAB. John Wiley & Sons, 2014.

Thank you!